# Intro to Graph Theory: Exploring the Four Color Problem With Math Models

Splash @UCLA 2024

Sarala

# Learning Objectives

Basic Graph Theory Vocabulary
Types of Graph
Applications of Graph Theory
Graph Coloring

Win a UCLA Keychain

# Icebreaker Would you rather...

lose your sense of smell or your ability to taste?

only be able to whisper or only be able to shout?

always have to tell the truth or never be able to speak again?

never be able to take a shower again or never be able to take a bath again?

spend the rest of your life without music or without movies?

spend a year on an island with someone who never stopped talking or completely alone?

live in a world where robots or aliens ruled the world?

live in a world where robots or aliens ruled the world?

never get angry or never get jealous?

rather know a little about numerous topics or know a lot about a few specific topics?

let a random person cut your hair or let a random person color your hair?

have the power to fall asleep on command or the power to wake up on command?

give up pizza or give up coffee forever?

be able to control fire or be able to control water?

have a free day off on Friday or Monday?

have excellent cooking skills or excellent organizing skills?

let a random person cut your hair or let a random person color your hair?

be great at something nobody cares about or average at something everyone cares about?

never have to do laundry or never have to do dishes again?

have all your food overseasoned or underseasoned?

be born with the head of a horse or the feet of a duck?

be able to mind control people or be able to put words in people's mouths?

have a car that always has fuel or a car that never needs repairs?

only be able to drink coffee or never be able to drink it again?

be able to mind control people or be able to put words in people's mouths?

eat a delicious meal with no utensils or a mediocre meal with utensils?

work in a high-paying job that you hate or in a low wage job you love?

never have a car or never have a pet?

be an infamous villain or an unknown superhero?

be in a room for a week with only one movie to watch or only one book to read?

# Why Model Math?

"Students who use accurate visual representations are six times more likely to correctly solve mathematics problems than are students who do not use them."

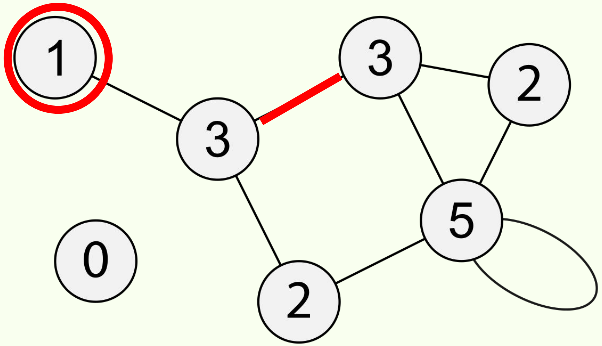
(Boonen, van Wesel, Jolles, & van der Schoot, 2014)

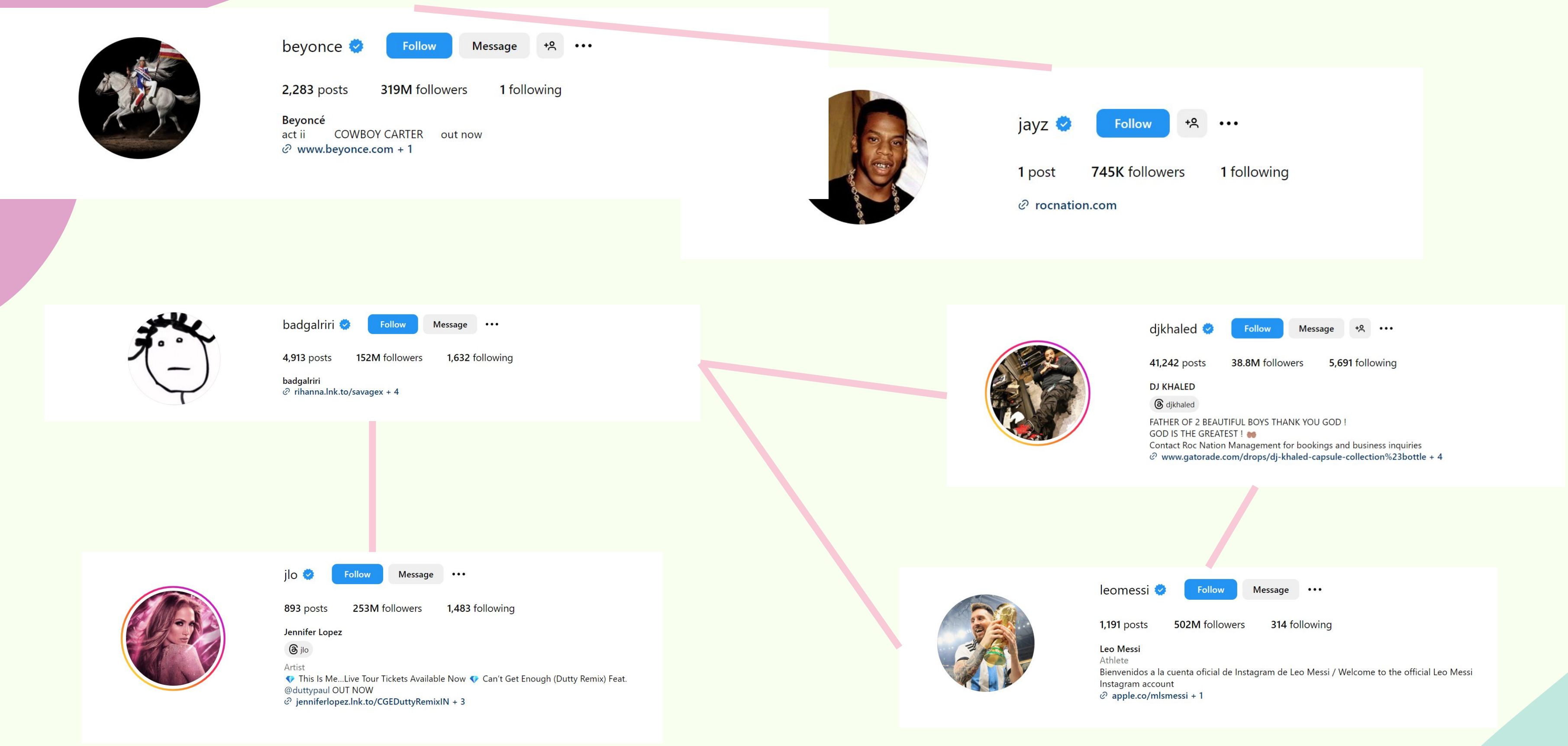
# What is a Graph

Data structures show relationships in computer science, biology, linguistics, and between many objects like web pages, programs, places, and people

Vertices can represent **objects** like courses or social media accounts.

Edges represent **relationships** like an order of courses or a social media friendships.




beyonce Follow Message
2,283 posts 319M followers 1 following
Beyoncé
act ii COWBOY CARTER out now
www.beyonce.com + 1
jayz Follow
1 post 745K followers 1 following
rocnation.com
badgalriri Follow Message
4,913 posts 152M followers 1,632 following
badgalriri
rihanna.lnk.to/savagex + 4
djkhaled Follow Message
41,242 posts 38.8M followers 5,691 following
DJ KHALED
djkhaled
FATHER OF 2 BEAUTIFUL BOYS THANK YOU GOD !
GOD IS THE GREATEST !
Contact Roc Nation Management for bookings and business inquiries
www.gatorade.com/drops/dj-khaled-capsule-collection%23bottle + 4
jlo Follow Message
893 posts 253M followers 1,483 following
Jennifer Lopez
jlo
Artist
This Is Me...Live Tour Tickets Available Now Can't Get Enough (Dutty Remix) Feat. @duttypaul OUT NOW
jenniferlopez.lnk.to/CGEDuttyRemixIN + 3
leomessi Follow Message
1,191 posts 502M followers 314 following
Leo Messi
Athlete
Bienvenidos a la cuenta oficial de Instagram de Leo Messi / Welcome to the official Leo Messi Instagram account
apple.co/mlsmessi + 1

# Data Structure

| | Jay Z | Beyonce | Rihanna | J Lo | DJ Khaled | Lionel Messi |
|---|---|---|---|---|---|---|
| Jay Z | | | | | | |
| Beyonce | | | | | | |
| Rihanna | | | | | | |
| J Lo | | | | | | |
| DJ Khaled | | | | | | |
| Lionel Messi | | | | | | |

# Vocabulary

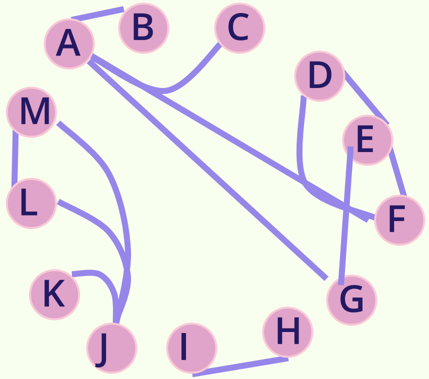
A **graph** is a set of vertices and edges that connect them

The precise way to represent this graph is to identify its *set of vertices V*:

*V* = {A, B, C, D, E, F, G, H, I, J, K, L, M},

and its *set of edges E* between these vertices:

*E* = {AG, AB, AC, LM, JM, JL, JK, ED,  FD, HI, FE, AF, GE}.



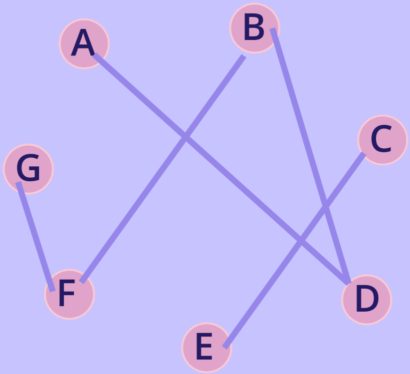

A and B are *incident* vertices
you get the point

# Vocabulary

What does this graph look like?

$V$ = {A, B, C, D, E, F, G}

$E$ = {AD, BD, FB CE, FG}.

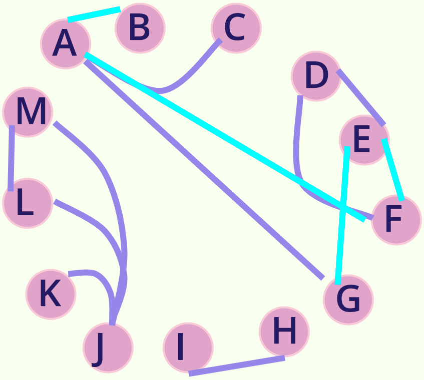


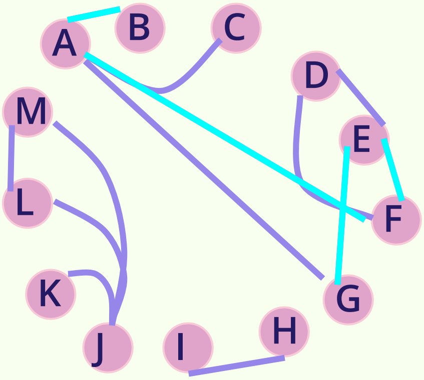
# Vocabulary

Path from vertex x to y in a graph is a list of vertices, in which successive vertices are connected by edges in the graph.

For example, BAFEG is path from B to G in the graph above.

A **simple path** has no vertex repeated.  For example, BAFEGAC is not a simple path.



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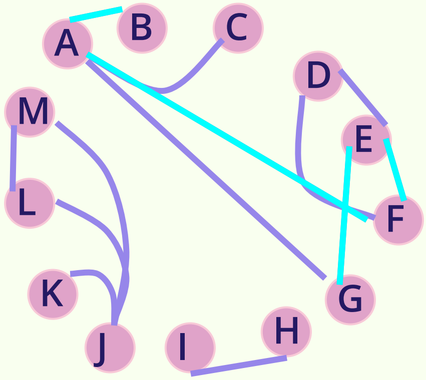
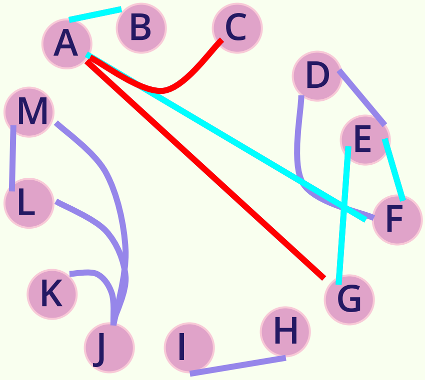


# Types of Graphs

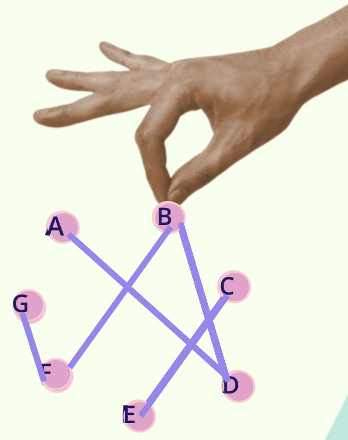
A graph is **connected** if there is a path from every vertex to every other vertex in the graph.  Intuitively, if the vertices were physical objects and the edges were strings connecting them, a connected graph would stay in one piece if picked up by any vertex.

A graph which is **not connected** is made up of connected components.  For example, the graph above has three connected components: {I, H}, {J, K, L, M} and {A, B, C, D, E, F, G}.

A **complete graph** has edges between every pair of vertices

An **empty graph** has no edges and just vertices

A **planar graph** has no edges that overlap



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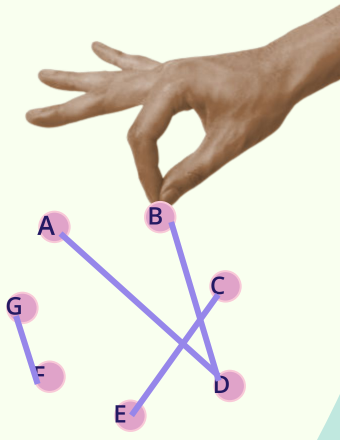


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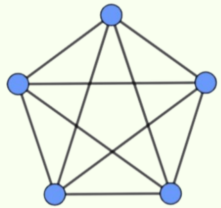

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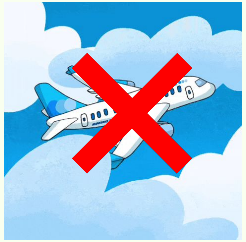
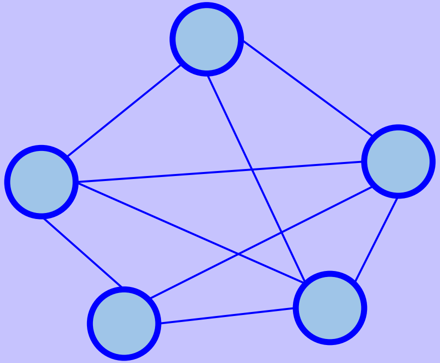
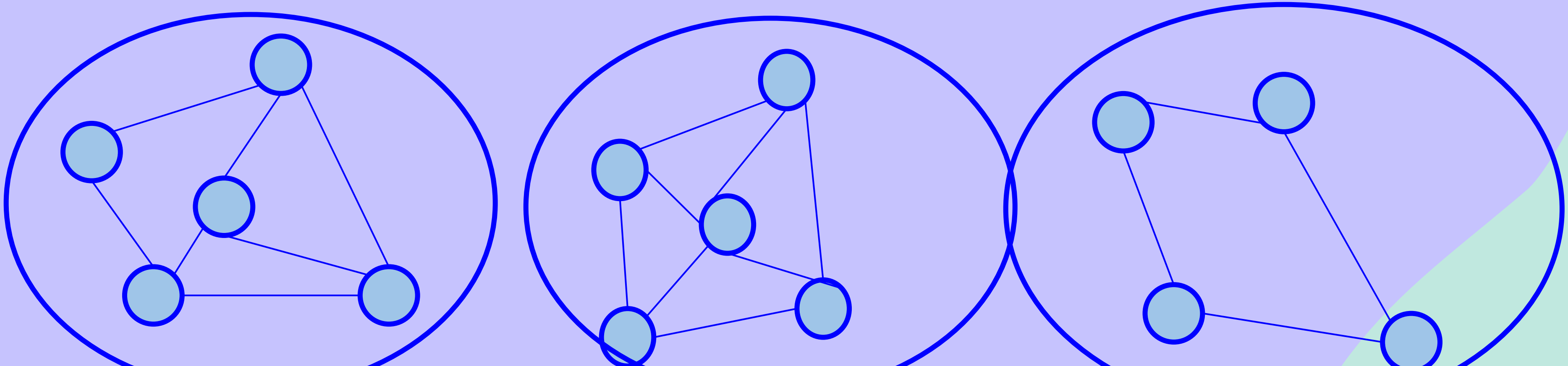


# Which Graphs Are Planar?

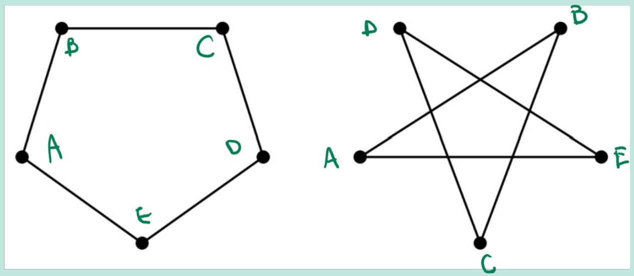




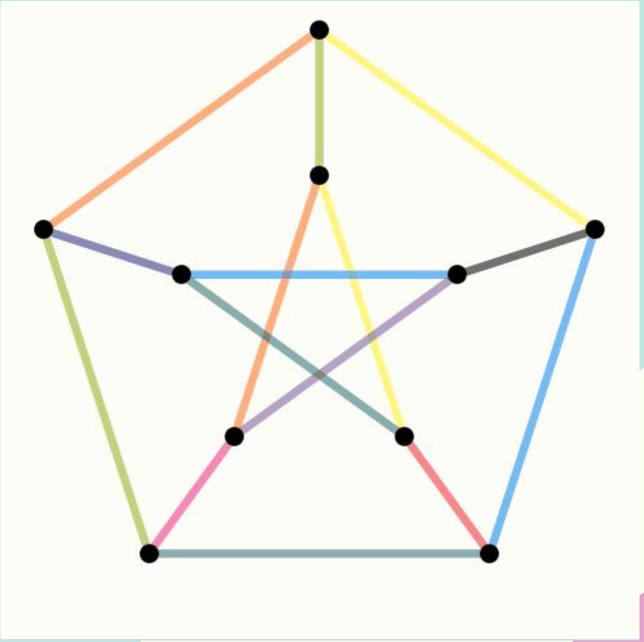
# Isomorphisms

Different models that represent graphs with the same set of vertice and edges are **isomorphisms**

Graphs $G_1$ and $G_2$
*V* = {A,B,C,D,E}
*E* = {AB, BC, CD, DE, EA}




$G_1$ $G_2$



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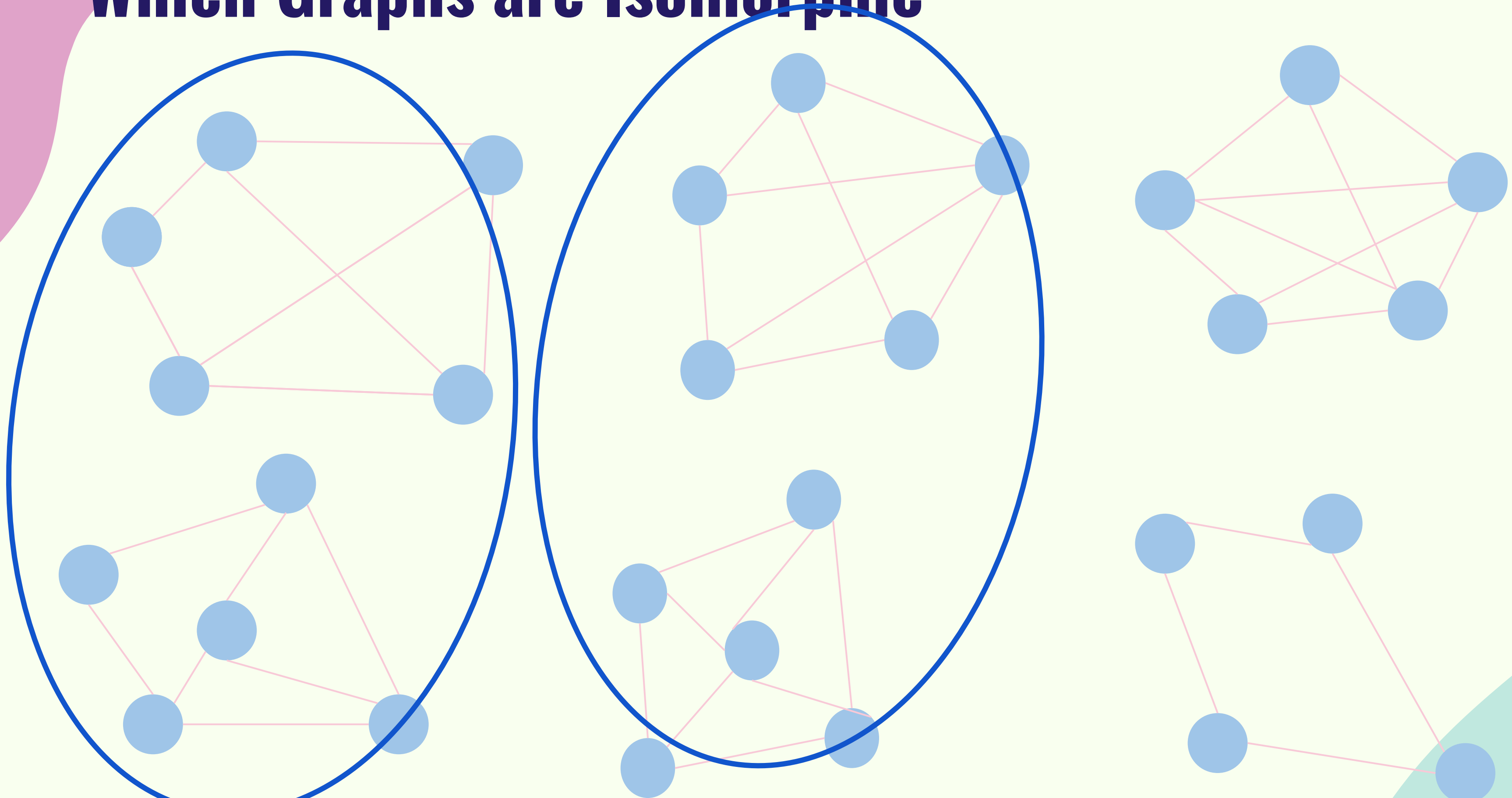
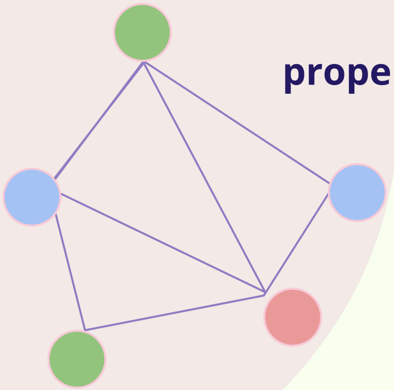


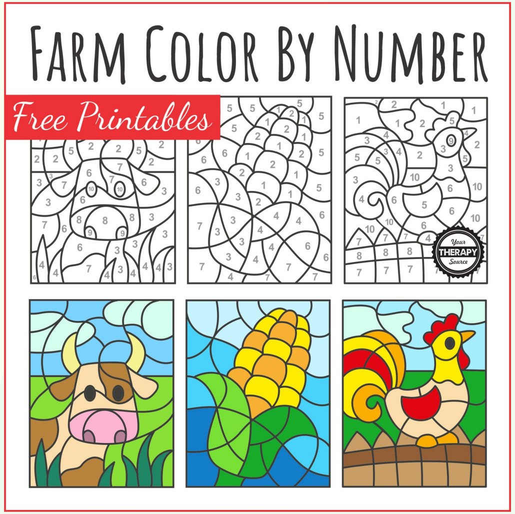
# Graph Coloring

Let G be a graph and C be a set of colors,

for example C = {black, white} or C = {1, 2}

A **proper coloring** of G by C is to assign a color from C to every vertex, such that in every edge vw, the vertices v and w have different colors.

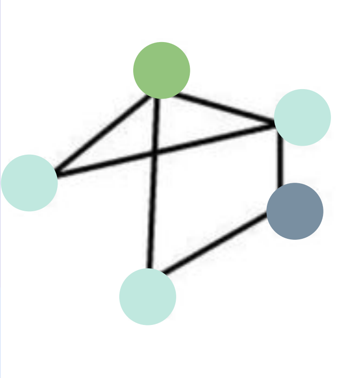


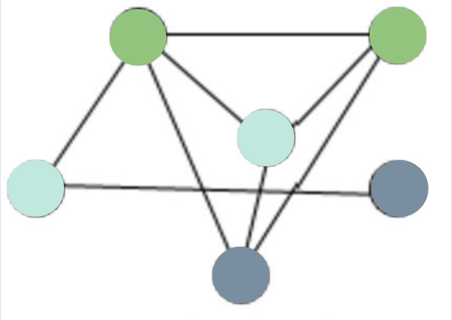


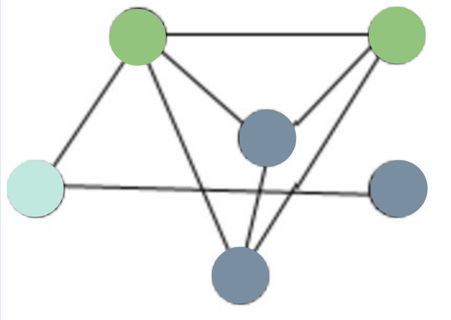

**proper 3-coloring**

**G is k-colorable**

# Which Graph is Has Proper Coloring?







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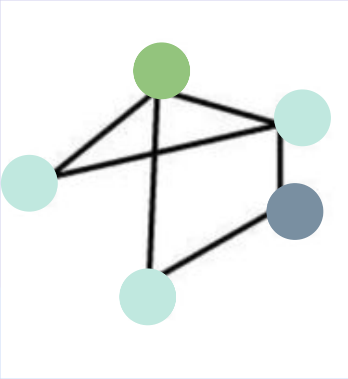
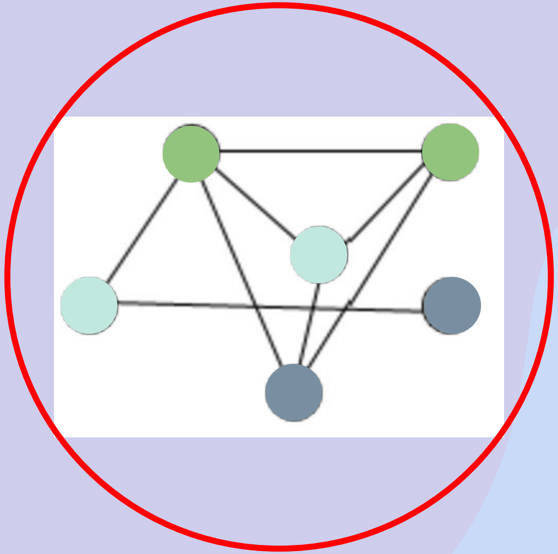
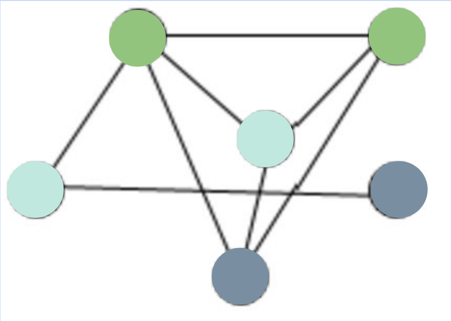
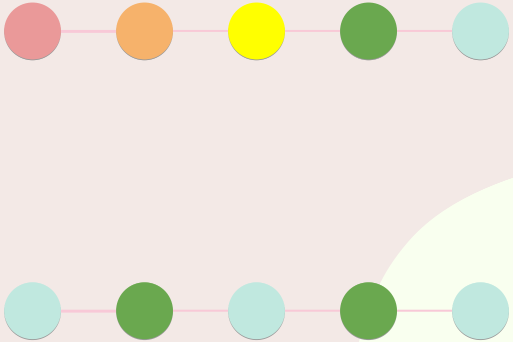
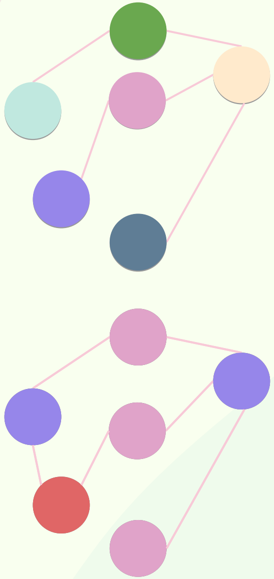






# What is the max # of colors in a properly colored graph?



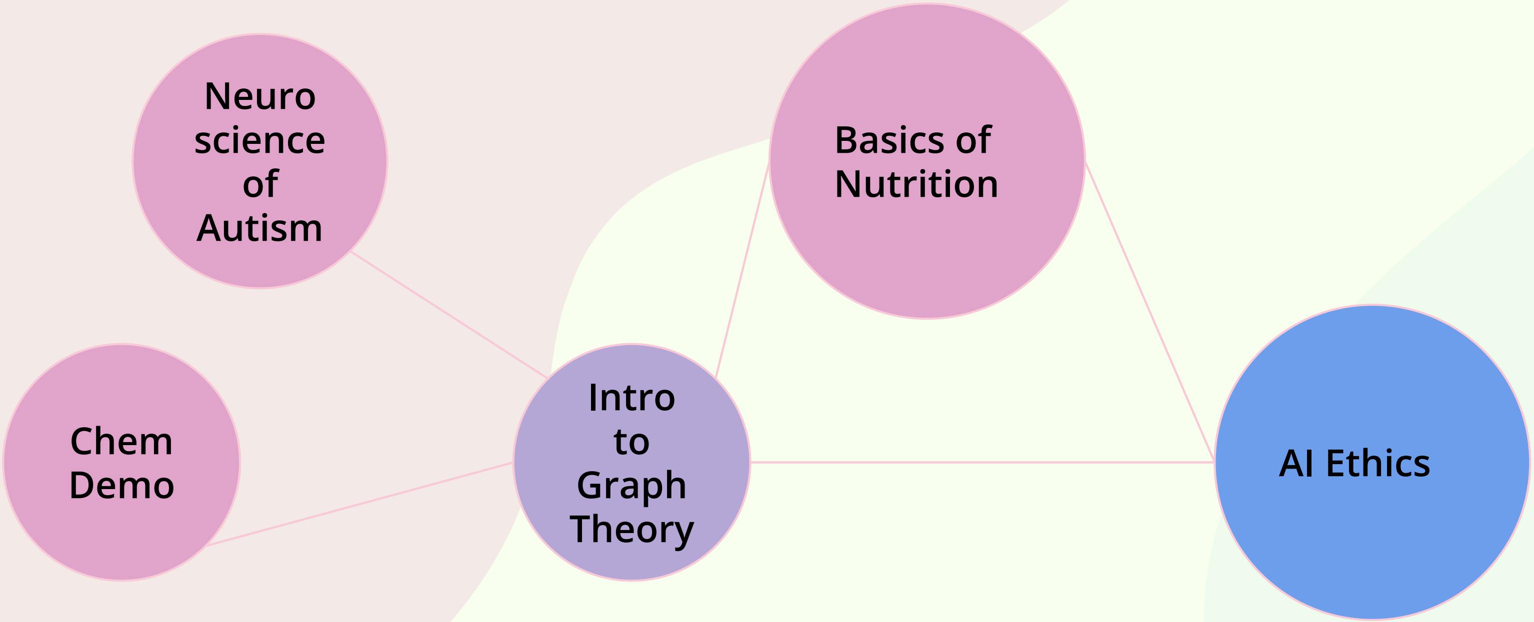


## What is the <u>min</u> # of colors in a properly colored graph?

## this number is known as the *chromatic number*

# Applications of Graph Coloring: Time Table

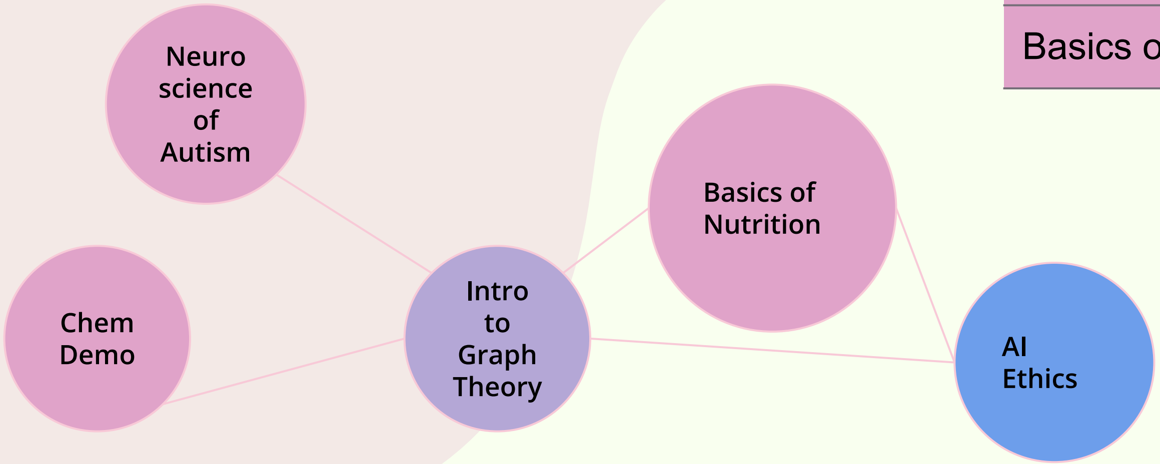
creating a time table with classes or exams that partially conflict



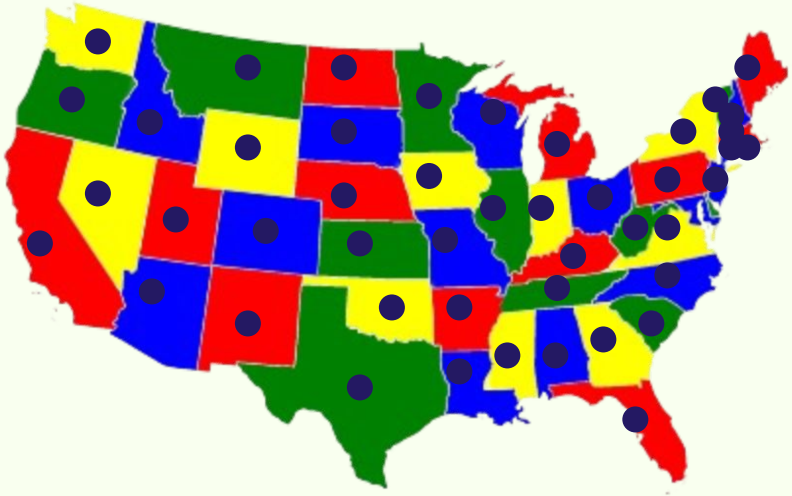
# Applications of Graph Coloring: Time Table

creating a time table with classes or exams that partially conflict

| Time Slot 1 | Time Slot 2 | Time Slot 3 |
|---|---|---|
| Intro to Graph Theory | Chem Demo | AI Ethics |
| | Neuroscience of Autism | |
| | Basics of Nutrition | |

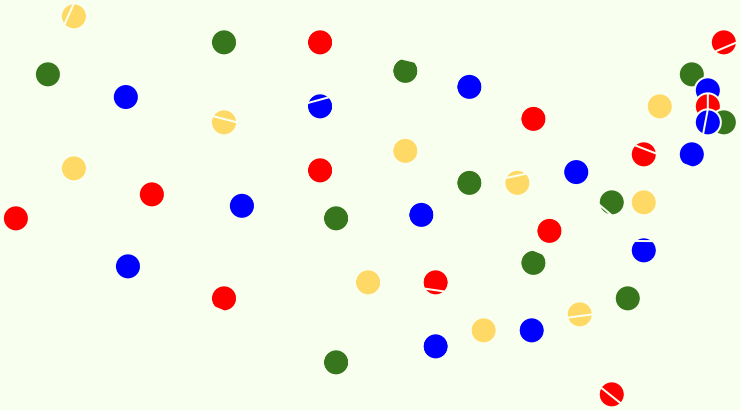


# Applications of Graph Coloring: Map Coloring



Each state represents a vertex

# Applications of Graph Coloring: Map Coloring



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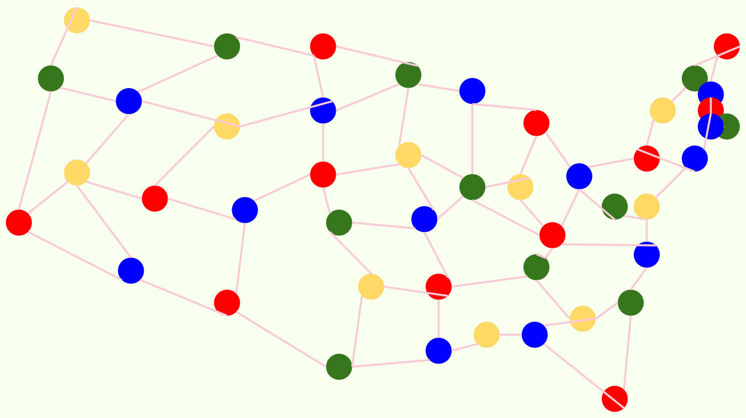
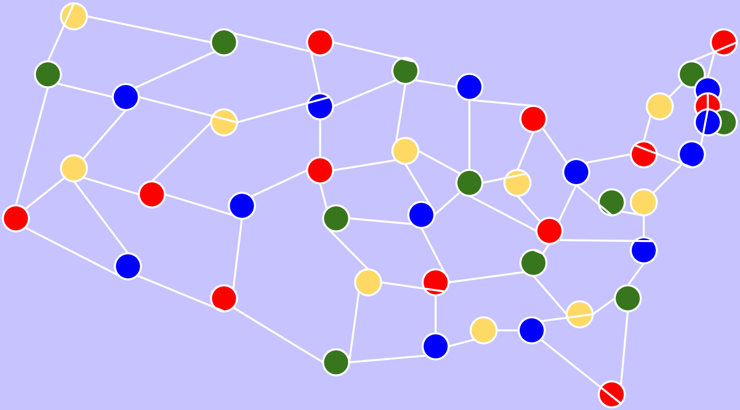
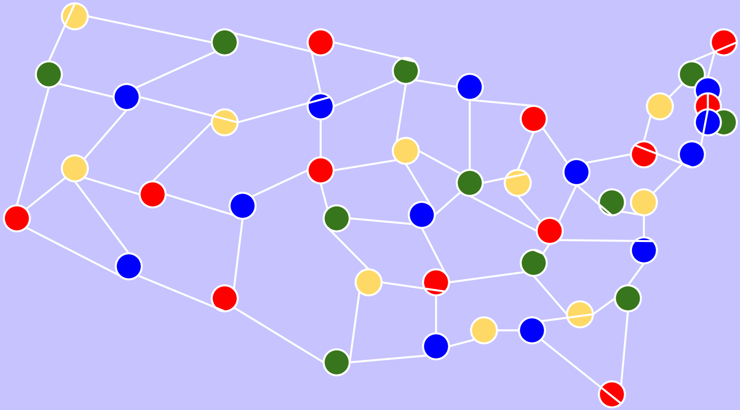


Each state represents a vertex
Each edge represents a border
And you have a map graph!

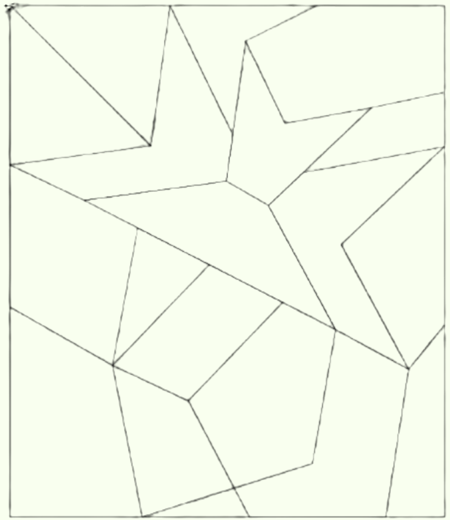
# Are Map Graphs Planar or Nonplanar?

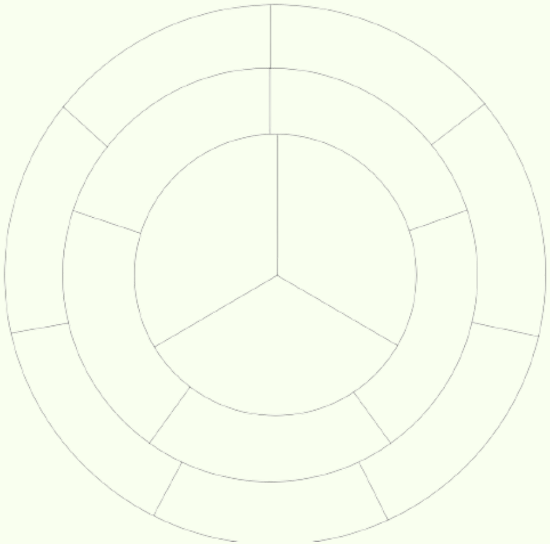


# What is the Chromatic Number of Map Graphs?



# Practice Graph Coloring Your Own Map!



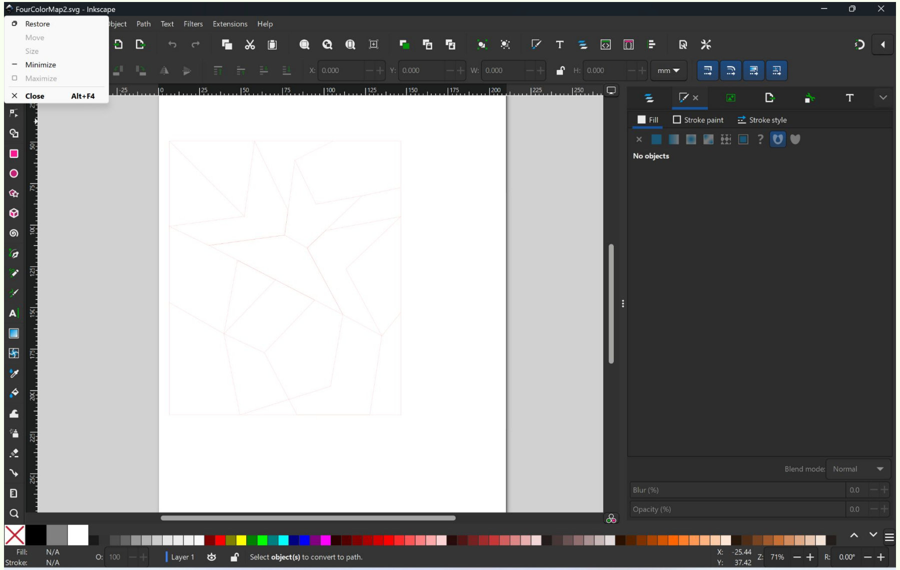


Break of into groups of 4
Objective:
Color the graph with minimum colors

# Laser Cutting Math Models

Laser cutting comprises of a few step:

1) Designing file (Inkscape or CorelDraw)
2) Saving as a .svg



# Laser Cutting Math Models

Laser cutting comprises of a few step:

1) Designing file (Inkscape or CorelDraw)
2) Saving as a .svg
3) Printing on a laser cutter in a makerspace (UCLA has two!)
    a) You can choose to raster, laser cut, or laser etch